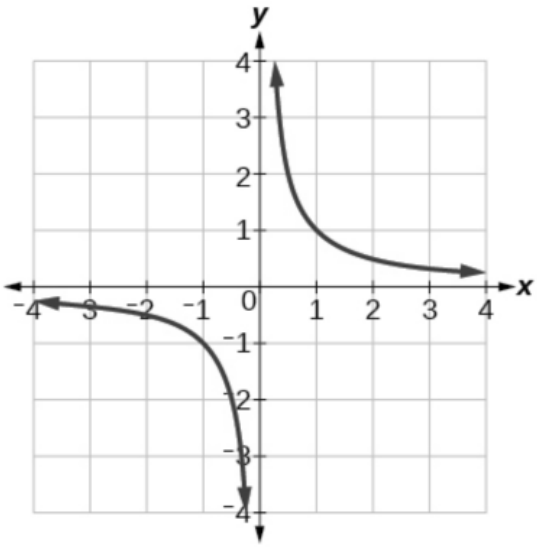
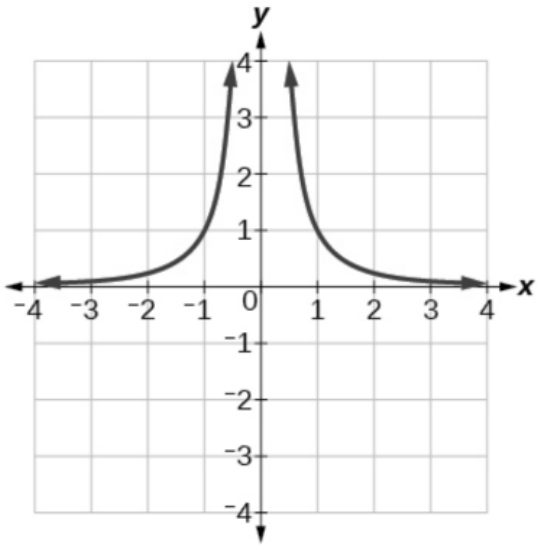
Graphing Rational Functions

Fraction



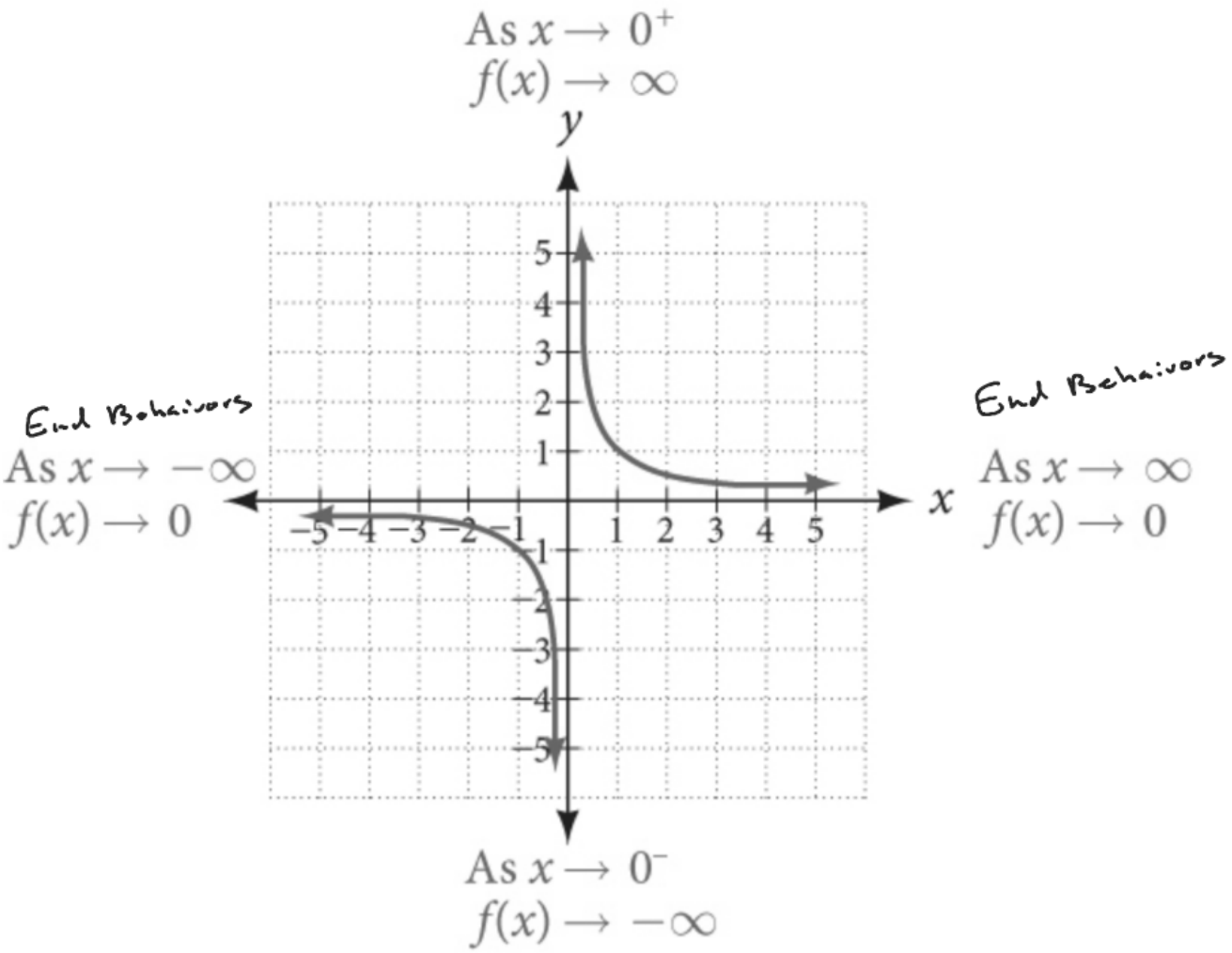
$f(x) = \frac{1}{x}$



$f(x) = \frac{1}{x^2}$

Arrow Notation

Symbol	Meaning
$x \to a^-$	x approaches a from the left ($x < a$ but close to a)
$x \to a^+$	x approaches a from the right ($x > a$ but close to a)
$x \to \infty$	x approaches infinity (x increases without bound)
$x \to -\infty$	x approaches negative infinity (x decreases without bound)
$f(x) \to \infty$	the output approaches infinity (the output increases without bound)
$f(x) \to -\infty$	the output approaches negative infinity (the output decreases without bound)
$f(x) \to a$	the output approaches a


As $x \to 0^+$
$f(x) \to \infty$
y
x
End Behaviors
As $x \to -\infty$
$f(x) \to 0$
End Behaviors
As $x \to \infty$
$f(x) \to 0$
As $x \to 0^-$
$f(x) \to -\infty$

What you will learn about:
Graphing Rational Functions

X-intercept:
Let y/f(x) = 0
and solve for x

Y-intercept
Let x = 0
Solve for y/f(x)

Rational Function
X-intercept
What value of x makes the top zero.

$x^2 - 36 = 0$
$(x-6)(x+6) = 0$
$x-6=0 \quad x+6=0$
$x=6 \quad x=-6$

Find the x-intercept(s) and y-intercept of each function.

A) $f(x) = x^2 - 36$

$0 = x^2 - 36$
$\sqrt{36} = \sqrt{x^2}$
$x = \pm 6$

$f(x) = 0^2 - 36$
$= -36$

B) $f(x) = \frac{x-5}{x+3}$

X-intercept $x-5=0$
$x=5$ $(5,0)$

Y-intercept: $f(0) = \frac{0-5}{0+3}$
$(0, -\frac{5}{3})$

C) $f(x) = \frac{x}{x+6}$

X-intercept (0,0)

$f(0) = \frac{0}{0+6} = \frac{0}{6}$

Y-intercept (0,0)

D) $\frac{x^2+4}{x+2} = f(x)$

X-intercept $x^2+4 = 0$
$x^2 = -4$
None

Y-intercept
$f(0) = \frac{0^2+4}{0+2} = \frac{4}{2}$
$= 2$
$(0,2)$

E) $f(x) = \frac{x^2-3x-10}{x}$

X-intercepts:
$x^2 - 3x - 10 = 0$
$(x+2)(x-5) = 0$
$x+2=0 \quad x-5=0$
$x=-2 \quad x=5$
$(-2,0) \quad (5,0)$

Y-intercept
$f(0) = \frac{0^2-3(0)-10}{0}$
$= -\frac{10}{0}$
None

$\frac{a}{b} = 0$

Domain:
X-values
Input

Point(s) of
Discontinuity

X-values that make the denominator zero.

Find the domain of the function algebraically. Support your answer graphically

A) $f(x) = x^2 - 9$

D: $(-\infty, \infty)$

B) $f(x) = \frac{1}{x+5}$

$x+5=0$

$x=-5$

D: $(-\infty,-5) \cup (-5,\infty)$

C) $f(x) = \frac{x}{x^2+2x-3}$

$x^2+2x-3=0$

$(x+3)(x-1)=0$

$x+3=0 \quad x-1=0$

$x=-3 \quad x=1$

D: $(-\infty,-3) \cup (-3,1) \cup (1,\infty)$

D) $f(x) = \frac{3}{x} + \frac{7}{x-1}$

P.O.D $x=0,1$

D: $(-\infty,0) \cup (0,1) \cup (1,\infty)$

E) $f(x) = \frac{x+6}{x^2-36}$

$x^2-36=0$

$(x+6)(x-6)=0$

$x+6=0 \quad x-6=0$

$x=-6 \quad x=6$

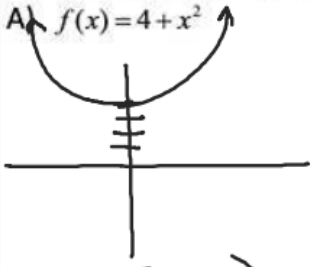
$(-\infty,-6) \cup (-6,6) \cup (6,\infty)$

Range
y-values
Output

Graph on Calculator and look at Graph

Determine the range of the function

A) $f(x)=4+x^2$



R: $[4,\infty)$

C) $f(x)=\frac{x^2}{4-x^2}$

R: $(-\infty,-1)\cup[0,\infty)$

D) $f(x)=\frac{3-2x^2}{4+x^2}$

R: $(-2,3/4]$

Graph the function and tell whether or not the function has a point of discontinuity at x = 0. If there is a discontinuity, tell whether the discontinuity is removable (Hole) or non-removeable (Vertical Asymptote).

A) $f(x)=\frac{5}{x}$

B) $f(x)=\frac{x^2+x}{x}$

C) $f(x)=\frac{|5x|}{x}$

D) $f(x)=\frac{2x}{x-4}$